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LETTER TO THE EDITOR

Spin-1 ferromagnetic chain in random anisotropy fields

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Abstract. The magnetization and susceptibility of the spin-1 ferromagnetic chain with random anisotropy fields has been investigated by Trotter–Suzuki decomposition and transfer matrix methods. The results indicate a zero temperature magnetization with a susceptibility exponent close to the spin wave value of 2.

The random field anisotropy mould was introduced by Harris, Plischke and Zuckerman (HPZ) [1] to describe amorphous magnets which are alloys of rare earths and transition metal elements. In this model each spin has a local random easy 2-fold axis and is described by the Hamiltonian

$$H=-\left(J\sum_{\langle ij\rangle} s_i\cdot s_j+D\sum_i (n_i\cdot s_i)^2+h\sum_i s_i^z\right). \tag{1}$$

In the above $\{n_i\}$ are random unit vectors denoting the directions of the easy axes and $\{s_i\}$ are spins. The original formulation of HPZ was in the language of quantum mechanics where $\{s_i\}$ are local angular momentum operators. (The actual Hamiltonian of HPZ differs in form from 1 as n_i was taken as the local axis of quantization.) However, investigations have almost exclusively been on the classical analogue of equation 1 where $\{s_i\}$ have been replaced by classical spins. These investigations have been for classical X–Y and Heisenberg spins and finite D, and also for the limit $D\to\infty$ when the Hamiltonian maps into an equivalent Ising system.

Within the classical framework the model of HPZ has had rather a checkered history with differing understanding of its critical properties being current at different times.

In three dimensions early numerical work by Harris and Zobin [2] supported the interpretation that for finite D ferromagnetism was destroyed and is replaced by a glassy type phase. Subsequent investigation by Harris and Sung [3] using Monte Carlo simulation overturned this conclusion with evidence suggesting the stability of the ferromagnetic phase. However, Pelcovits *et al* [4] in work produced at about the same time suggested the instability of the ferromagnetic phase in less than four dimensions. Other work [5] concluded that the three dimensional system was qausiferromagnetic, that is zero ferromagnetic order parameter with infinite correlation length. In three dimensions numerical work has been done on this model in the limit $D\to\infty$ when the spins become Ising like. In this limit large cell renormalization [6] has suggested no ferromagnetism and no quasiferromagnetism. Monte Carlo work [7] indicated spin glass type order only. There are also general energy balance arguments suggesting the destruction of ferromagnetism for all D is less than four dimensions [8]. Thus a range of suggestions exists.

Recently Reed [9] has examined this system using Monte Carlo simulation and finite size scaling for two component X–Y spins and for both 2-fold and 3-fold anisotropy fields and $D=J$. The results indicate little difference between the 2-fold and 3-fold case. Both systems appeared critical at a temperature close to the transition temperature of the pure system and with an exponent ν which again within numerical uncertainty is almost identical to the pure system. All this would argue for the irrelevance of D at the ferromagnetic transition. No evidence of quasiferromagnetism was found for 2 or 3-fold anisotropy. However, the magnetization was not measured in this simulation and so it is not possible to be certain about the nature of the low temperature state but the evidence argued in favour of ferromagnetism.

Recently Fisch [10] has also investigated the problem for 2 and 3-fold anisotropy by sophisticated Monte Carlo methods in three dimensions for X–Y spins but in the case where $D\rightarrow\infty$ when the system becomes Ising like. The results of that simulation indicated ferromagnetism for 3-fold axes but quasiferromagnetism for 2-fold axes.

From the above it would be difficult to be certain about the critical properties of this model in three dimensions, however, recent evidence would probably indicate the survival of ferromagnetism in three dimensions for X–Y spins.

In two dimensions the picture is perhaps a little clearer following the careful renormalization group calculation of Cardy and Ostlund [11]. For general p-fold anisotropy and X–Y spins their calculation predicted the irrelevance of the anisotropy field for all $p\geqslant 3$ with the case $p=4$ being special. Monte Carlo [12] calculation has confirmed the overall picture. However for, $p=2$ which is outside the domain of Cardy and Ostlund's results, but is the appropriate value for the model of HPZ, Monte Carlo simulation indicated the destruction of the Kosterlitz–Thouless phase and the existence of a glassy phase. This conclusion has received support from other recent Monte Carlo simulation [13].

Even in one dimension there is room for doubt. For finite D, Serota and Lee [14] were unable to find a groundstate with zero magnetization. This result does not quite conform to the numerical work of Dickinson and Chudnovsky [15]. However, this latter work was done for thermodynamically typical low-energy states which are appropriate for comparison with experiment but may not be the true groundstate. What can be certain is that in the limit $D\rightarrow\infty$ the model reduces to the one-dimensional Ising model with random symmetric bonds and this is certainly not ferromagnetic in the groundstate.

In order to attempt to illuminate the properties of the model of HPZ and also to revert to the original quantum nature of the Hamiltonian an investigation has been made for the case of spin-1 operators in one dimension for the case $J=D$. It is hoped that work in one dimension can form a basis for understanding in higher dimensions. The quantities studied were the magnetization and the susceptibility.

The method used for the investigation was the Trotter–Suzuki method [16] using a checkerboard decomposition [17]. This results in the original quantum chain being transformed into a $2m\times N$ classical rectangular lattice with four spin interactions in alternative rectangles. Here N is the length of the original chain and $2m$ is the dimension in the Trotter direction. This procedure is well known from quantum Monte Carlo simulations. However, to avoid the 'sign' problem which creates difficulties at low temperature this method is not used and the transfer matrix approach of Binder and Morgenstern [18] applied instead. This avoids the 'sign'

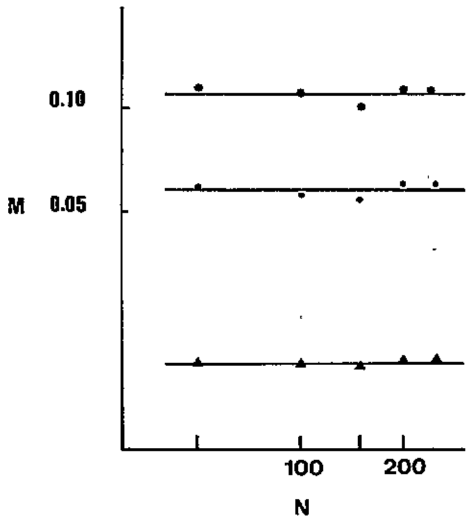

Figure 1. Magnetization M against system size N on a log–log plot. (Δ) $T=0.7J$. (●) $T=0.4J$. ($*$) $T=0.3J$.

problem. In fact since the Hamiltonian is hermitian rather than just symmetric the 'sign' problem would be worse in this case as the matrix elements would be complex.

The free energy F was calculated and the magnetization M and susceptibility χ calculated using

$$M=-\frac{\partial F}{\partial h}\bigg|_{h\to 0_+}$$

$$\chi=-\frac{\partial^2 F}{\partial h^2}\bigg|_{h\to 0_+}.$$

In practice the differentials were calculated using finite differences.

Results were calculated for system sizes ranging from $N=50$ to $N=250$ and for $m=4, 5, 6$. The error in thermodynamic quantities induced by the methods is believed to be a polynomial in $({}^{J}/_{Tm})^2$ where T is the temperature. This restricts the range of temperature at which useful results can be obtained to $({}^{J}/_{Tm})^2<1$. This behaviour of the error in the thermodynamic function has been checked by plotting the results against a linear function of $({}^{J}/_{Tm})^2$. The results were as expected; at high temperature the goodness of fit parameter was virtually zero, increasing as the temperature is lowered as higher-order terms in the polynomial begin to have an effect. To account for this and to accommodate the slight convex nature of the data the fit was made to a quadratic for all temperatures in the range $0.3\leqslant T/J\leqslant 0.8$.

For each value of N only one realization of the anisotropy field directions was used. This was because sample to sample fluctuations were not pronounced and to allow all other extrapolations to be done on noiseless data.

Results for the magnetization are shown in figure 1 as a function of N and for varying temperatures. No tendency for M to diminish as N increases is detectable.

This is entirely consistent with a first-order transition to a non-zero magnetization at $T=0$.

The susceptibility has been fitted to the form

$$\chi = T^{-\gamma}.$$

In the range $0.3 \leqslant T/J \leqslant 0.8$ the values of γ all lay in the range 1.978 to 2.187 which is entirely consistent with a value of $\gamma=2$, the spin wave value.

As with any procedure which is not exact the domain of accuracy of these results could always be challenged. In particular it might be suggested that the temperatures are not sufficiently low to exhibit the asymptotic behaviour. On the other hand the procedure does not have any uncontrollable errors.

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